CCGPS Frameworks Student Edition

Mathematics

6th Grade
Unit 3: Expressions

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"Making Education Work for All Georgians"

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Unit 3
Expressions

TABLE OF CONTENTS

OVERVIEW

In this unit students will:

- Represent repeated multiplication with exponents
- Evaluate expressions containing exponents to solve mathematical and real world problems
- Translate verbal phrases and situations into algebraic expressions
- Identify the parts of a given expression
- Use the properties to identify equivalent expressions
- Use the properties and mathematical models to generate equivalent expressions

Students working with expressions and equations containing variables allows for them to form generalizations. Students should think of variables as quantities that vary instead of as letters that represent set values. When students can work with expressions involving variables without the focus on a specific number or numbers that the variable may represent they can focus on the patterns that occur. It is these patterns that lead to generalizations that lay the foundation for their future work in algebra

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics especially with respect to fluency.

KEY STANDARDS

Apply and extend previous understandings of arithmetic to algebraic expressions.

MCC6.EE.1 Write and evaluate expressions involving whole-number exponents.

MCC6.EE.2 Write, read, and evaluate expressions in which letters stand for numbers.

MCC6.EE.2a Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as 5-y.*

MCC6.EE.2b Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2(8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.

MCC6.EE.2c Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to

specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = \frac{1}{2}$.

MCC6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

MCC6.EE.4 Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them.) *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

RELATED STANDARDS

MCC6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because ¾ of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share ½ lb. of chocolate equally? How many ¾ - cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length ¾ mi and area ½ square miles?*

<u>Compute fluently with multi-digit numbers and find common factors and multiples</u>

MCC6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

MCC6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

MCC6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4(9 + 2).

STANDARDS FOR MATHEMATICAL PRACTICE:

1. Make sense of problems and persevere in solving them. Students make sense of expressions and formulas by connecting them to real world contexts when evaluating.
2. Reason abstractly and quantitatively. Students contextualize to understand the meaning of the number or variable as related to the problem and decontextualize to manipulate symbolic representations by applying properties of operations.
3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding the equivalence of expressions and the use of variable expressions to represent real-world situations.
4. Model with mathematics. Students form expressions from real world contexts. Students use algebra tiles to model algebraic expressions.
5. Use appropriate tools strategically. Students determine which algebraic representations are appropriate for given contexts.
6. Attend to precision. Students use the language of real-world situations to create appropriate expressions.
7. Look for and make use of structure. Students apply properties to generate equivalent expressions. They interpret the structure of an expression in terms of a context. Students identify a "term" in an expression.
8. Look for and express regularity in repeated reasoning. Students can work with expressions involving variables without the focus on a specific number or numbers that the variable may represent. Students focus on the patterns that lead to generalizations that lay the foundation for their future work in algebra. Students work with the structure of the distributive property $2(3x + 5) = 6x + 10$.

ENDURING UNDERSTANDINGS

- Variables can be used as unique unknown values or as quantities that vary.
- Exponential notation is a way to express repeated products of the same number.
- Algebraic expressions may be used to represent and generalize mathematical problems and real life situations
- Properties of numbers can be used to simplify and evaluate expressions.
- Algebraic properties can be used to create equivalent expressions
- Two equivalent expressions form an equation.

CONCEPTS/SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Using parentheses, brackets, or braces in numerical expressions and evaluate expressions with these symbols.

- Write and interpret numerical expressions.
- Generating two numerical patterns using two given rules.
- Interpret a fraction as division
- Operations with whole numbers, fractions, and decimals

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

http://www.corestandards.org/Math/Content/mathematics-glossary/glossary

- **Algebraic expression**: A mathematical phrase involving at least one variable and sometimes numbers and operation symbols.

- **Associative Property of Addition**: The sum of a set of numbers is the same no matter how the numbers are grouped.

- **Associative Property of Multiplication:** The product of a set of numbers is the same no matter how the numbers are grouped.

- **Coefficient:** A number multiplied by a variable in an algebraic expression.
- **Commutative Property of Addition:** The sum of a group of numbers is the same regardless of the order in which the numbers are arranged
- **Commutative Property of Multiplication:** The product of a group of numbers is the same regardless of the order in which the numbers are arranged.
- **Constant:** A quantity that does not change its value.
- **Distributive Property:** The sum of two addends multiplied by a number is the sum of the product of each addend and the number.
- **Exponent:** The number of times a number or expression (called base) is used as a factor of repeated multiplication. Also called the power.
- **Like Terms:** Terms in an algebraic expression that have the same variable raised to the same power. Only the coefficients of like terms are different.
- **Order of Operations:** The rules to be followed when simplifying expressions.
- **Term:** A number, a variable, or a product of numbers and variables.
- **Variable:** A letter or symbol used to represent a number or quantities that vary

MISCONCEPTIONS

- The mnemonic PEMDAS can mislead students into thinking that addition must come before subtraction and multiplication must come before division.
- Students fail to see juxtaposition (side by side) as indicating multiplication. For example, evaluating 3x as 35 when x = 5 instead of 3 times 5 = 15. Also, students may rewrite 8 – 2a as 6a.
- Students also miss the understood "1" in front of a lone variable like a or x or p. For example, not realizing that 4a + a is 5a.
- Many of the misconceptions when dealing with expressions stem from the misunderstanding/reading of the expression. For example, knowing the operations that are being referenced with notation like x^3, $4x$, $3(x + 2y)$ is critical. The fact that x^3 means (x)(x)(x) which is x times x times x, not $3x$ or 3 times x; $4x$ means 4 times x or $x + x + x + x$, not forty-something.

Formative Assessment Lessons (FALs)

Formative Assessment Lessons are intended to support teachers in formative assessment. They both reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.
More information on types of Formative Assessment Lessons may be found in the Comprehensive Course Guide.

Name_____

TASK: EXPONENTS

You and a partner need a red and a green die. The red die will be the base and the green die will be the exponent. Take turns rolling the dice, record the base and the exponent. After you have recorded 10 numbers find the standard form of each of the exponents that you have written down. Race your partner and see who finishes first.

Base	Exponent	Exponential Form	Standard Form	Value

Name_____

TASK: RULES FOR EXPONENTS

1. With a partner determine if the following expressions are equivalent

a. $2^2 \cdot 3^2 - 2^3 - 1$

b. $2^2 \cdot (3^2 - 2^3) - 1$

c. $(2 \cdot 3)^2 - 2^3 - 1$

2. Write an expression of your own using all the operations as well as exponents

3. Rewrite the expression using grouping symbols to give a different answer.

4. Explain the Order of Operations and how it is useful in solving mathematical and real world problems

Name_____

TASK: CONJECTURES ABOUT PROPERTIES

With a partner look at the following sets of number sentences and determine if what you observe would be true for all numbers. Create statements with words about what you observe in each set of number sentences then write the number sentences using variables to represent numbers.

12 + 0 = 12 37 + 0 = 37 64 + 0 = 64	12 – 0 = 12 37 – 0 = 37 64 – 0 = 64	$12 \cdot 1 = 12$ $37 \cdot 1 = 37$ $64 \cdot 1 = 64$

$12 \div 1 = 12$ $37 \div 1 = 37$ $64 \div 1 = 64$	$12 \cdot 0 = 0$ $37 \cdot 0 = 0$ $64 \cdot 0 = 0$	12 ÷ 0 = 12 45 ÷ 0 = 45 64 ÷ 0 = 64

12(4 + 3) = 48 + 36 6(7 + 2) = 42 + 12 4(10 + 3) = 40 + 12	$12(4 - 3) = 48 - 36$ $6(7 - 2) = 42 - 12$ $4(10 - 3) = 40 - 12$

4 x 8 = (2 x 8) + (2 x 8)

8 x 16 = (4 x 16) + (4 x 16)

5 x 14 = (2.5 x 14) + (2.5 x 14)

4 + 8 = (2 +8) + (2 + 8)

8 + 16 = (4 +16) + (4 + 16)

5 + 14 = (2.5 + 14) + (2.5 + 14)

(32 + 24) + 16 = 32 + (24 + 16)

(450 + 125) + 75 = 450 + (125 + 75)

(33 + 17) + 3 = 33 + (17 + 3)

$6 \cdot (4 \cdot 3) = (6 \cdot 4) \cdot 3$

$10 \cdot (5 \cdot 2) = (10 \cdot 5) \cdot 2$

$(11 \cdot 2) \cdot 3 = 11 \cdot (2 \cdot 3)$

Name_____

TASK: WRITING EXPRESSIONS

1. Within your classroom, have the students find situations where they can role play to compare known and unknown quantities (e.g., Student A (Dory) and Student B (Colleen).For example Dory says, "I have two sisters." Colleen says, "I have Dory – 1 sisters." Dory says, "You have d – 1 sister. You have one sister.") Make sure all operations are included. Write expressions here.

2. Give each pair of students an expression such as x + 957. Challenge them to find a way to evaluate each expression for x = 35, 825, and 373. Then have the students write a real-life context for each expression. Write expressions here.

3. Write each word phrase as an algebraic expression.

6 less than 3t	the product of w and 8
r divided by 15	9 more than twice x
the quotient of 12 and x	the product of x and 6
the sum of three times a and 35	six times the sum of x and 8
a number, x, decreased by 9	a number increased by the quotient of x and 7
15 less than 4 times 11	a number, n, decreased by the difference of x and 7

4. Hannah is 3 years younger than Katie. Joey is twice as old as Hannah.
   Let k stand for Katie's age. Write an expression to represent Hannah's age. Using k, write an expression for Joey's age.

Name_____

TASK: WRITING AND EVALUATING EXPRESSIONS

Part I

Mr. Green's Math class is planning a trip to the IMAX Theater. It will cost $10 for the school bus and the price of a ticket is $13 dollars per student. What will determine the amount of money the class will have to make?

How will the number of students affect the price?

How will they know how much money they need to make?

What value varies in this example?

Write an expression to show the amount of money the class needs to make.

How much will it cost if 10 students attend?

How much will it cost if 17 students attend? Draw a model to represent this situation

Part II

For the first five problems read each carefully and write an expression that includes numbers and variables. Then, evaluate the expression using the numbers indicated. For the last five problems evaluate the expression for the numbers provided.

1. Mr. White drives 55 km a day for work. How many km will he drive in:
   a. 5 days?
   b. 8 days?
   c. 15 days?
   d. Write an expression to represent the number of km he will drive in d days

2. Sean's father is working on a crew that will build a skyscraper. He found out that each story is 13 ft tall. How tall, in feet, would the skyscraper be if it were:

   a. 55 stories?

   b. 65 stories?

   c. 75 stories?

   d. Write an expression to represent the height of a skyscraper with f stories

3. 55 figurines of a porcelain doll can be safely shipped in a case. A distributor is investigating to find which size box is the safest to hold the largest number of cases. How many figurines could be shipped in a box that could hold:

   a. 750 cases?

   b. 1000 cases?

   c. 1250 cases?

   d. Write an expression to represent the number of figurines that can be shipped in a box that holds c cases

4. The rental fee for a bike is $10 plus $3 for each hour the bike is used. How much will it cost if you rent the bike for:

   a. 1 hour?

   b. 8 hours?

   c. 1 day?

   d. Write an expression that represents the cost for h hours

5. A wireless service provider charges $29.99 per month for service plus $0.10 for each text message. How much will it cost if:

a. 35 text messages are sent?

b. 105 text messages are sent?

c. 217 text messages are sent?

d. Write an expression to represent the cost if t text messages are sent

6. The formula for finding the Volume of a rectangular prism can be stated as $V = l \times w \times h$, where l = length of the prism, w = width of the prism and h = height of the prism. What is the Volume of a prism with:

a. $l = 33$, $w = 47$, and $h = 15$?

b. $l = 22.5$, $w = 33.7$, and $h = 12.5$?

c. $l = 122.25$, $w = 50.75$, and $h = 16.5$?

7. The formula for finding the volume of a prism is V=Bh. What is the volume of the prism with:

   a. Area of the base is 16 cm and height is 2.4 cm?

   b. Area of the base is 12½ cm and height is 7cm?

   c. Area of the base is $3\frac{3}{4}$ cm and height is $3\frac{1}{5}$ cm?

8. The formula for finding the volume of a cube is $V = s^3$, where s is equal to the length of one side of the cube. What is the volume of the cube with side length of:

a. $\frac{1}{2}$ *inches*?

b. 26.4 *meters*?

c. 100 *feet?*

9. The formula for finding the surface area of a cube is $A = 6s^2$, where s is the length of one side of the cube. What is the surface area of a cube with side length of:

a. $\frac{1}{2}$ inches?

b. 26.4 meters?

c. 100 feet?

Formative Assessment Lesson: Laws of Arithmetic

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1358

TASK COMMENTS:

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Laws of Arithmetic*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=484&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1358

STANDARDS ADDRESSED IN THIS TASK:

MCC6.EE.3 Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3(2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6(4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

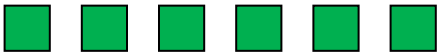
Standards for Mathematical Practice
This lesson uses all of the practices with emphasis on:
1. Make sense of problems and persevere in solving them.
5. Use appropriate tools strategically.
7. Look for and make use of structure.

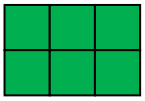
Name_____

TASK: ARE WE EQUAL

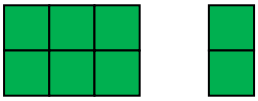
Numerical Expressions

To model whole numbers we use unit cubes. For example:

6  or we can arrange them as an array to show the factors



To model a numeric expression we would use groups of unit cubes

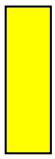
6 + 2 

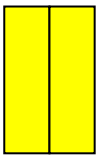
5 + 3 

1. Are these two numeric expressions equivalent?

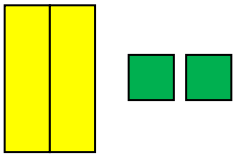
To model variable expressions we use the *x* and *y* tiles. They don't have to be called *x* and *y*. They could be *a* and *b* or *s* and *t* or *b* and *w*. The point is that they represent one of an unknown.

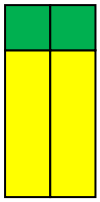
Algebraic Expressions

x 

2*x* 

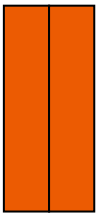
We have a rectangle that has dimensions of 2 and *x* or 2*x*

2*x* + 2 

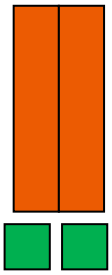
We can also arrange them in this way 

We can model these same examples with our *y*

y 

2*y* 

$2y + 2$



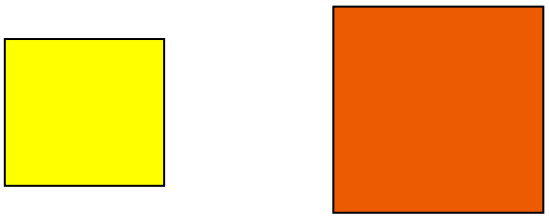
Right now we are not concerned with the value of *x* and *y*. We just want to know how many *x*'s or how many *y*'s.

2. Draw two different models of $2(x + 1)$?

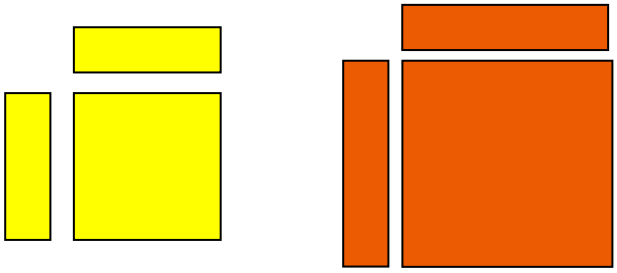
3. Draw a model to represent 2x + 2

4. Based on your two models what conclusion can you draw about 2(x + 1) and 2x + 2. Be sure to justify your conclusion.

Now let's look at two more tiles



They are called x^2 and y^2 because the length of the sides of each is equal to our x and y tiles



5. What would $2x^2$ look like?

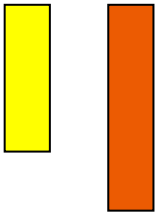
6. What would $2x^2 + 4$ look like?

7. Can you divide these into equal groups? Draw a model to represent this and write an algebraic expression to represent your model

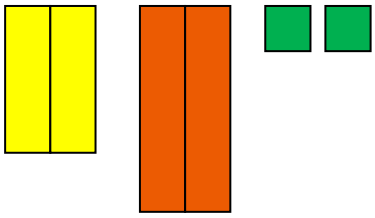
8. Based on the models what conclusion can you draw about $2x^2 + 4$ and $2(x^2 + 2)$? Be sure to justify your conclusion.

Let's try combining variables.

What would $x + y$ look like?



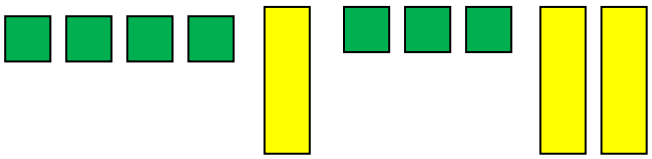
So, $2x + 2y + 2$ could be modeled



9. Is it possible to divide these into equal groups? Draw a model to represent this and write an algebraic expression to represent your model

10. Based on the models what conclusion can you draw about $2x + 2y + 2$ and $2(x + y +1)$. Be sure to justify your conclusion.

Let's look at this example: $4 + x + 3 + 2x$



11. Is there a way to rearrange things so it is a little neater? Draw a model to justify your answer

12. Can we write a new number sentence to represent this?

13. What conclusion can you draw about $4 + x + 3 + 2x$ and $3x + 7$?

With a partner, determine whether each of the following pairs of expressions are equivalent. Some of them may not be equivalent. Be sure to justify your conclusions

14. $6y + 12$ and $6(y + 2)$

15. $3x + y$ and $y + 3x$

16. $3x + 2$ and $3(x + 2)$

17. $5x^2 + 15$ and $5(x^2 + 3)$

18. $3y^2 + 6x^2$ and $3(y^2 + 2x^2)$

Now, find an equivalent expression or expressions for each of the following. Draw your representations. Write an equation to show the expressions are equal.

19. $3y^2 + 2 + 1$

20. $2y + y + 4 + 2$

21. $2y^2 + 4x^2$

22. $2 + 3x^2 + x + 2x + 1$

23. y + y + y + y