CCGPS Frameworks Student Edition

Mathematics

6th Grade

Unit 2: Rate, Ratio and Proportional Reasoning Using Equivalent Fractions

Dr. John D. Barge, State School Superintendent

"Making Education Work for All Georgians"

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Unit 2
Rate, Ratio, and Proportional Reasoning Using Equivalent Fractions

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OVERVIEW

In this unit students will:

- gain a deeper understanding of proportional reasoning through instruction and practice
- develop and use multiplicative thinking
- develop a sense of proportional reasoning
- develop the understanding that ratio is a comparison of two numbers or quantities
- find percents using the same processes for solving rates and proportions
- solve real-life problems involving measurement units that need to be converted

STANDARDS ADDRESSED IN THIS UNIT

KEY STANDARDS

Understand ratio concepts and use ratio reasoning to solve problems.

MCC6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

MCC6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$ (b not equal to zero), and use rate language in the context of a ratio relationship.

MCC6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

MCC6.RP.3a Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

MCC6.RP.3b Solve unit rate problems including those involving unit pricing and constant speed.

MCC6.RP.3c Find a percent of quantity as a rate per 100, (e.g. 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole given a part and the percent.

MCC6.RP.3d Use ratio reasoning to convert measurement units, manipulate and transform units appropriately when multiplying or dividing quantities.

RELATED STANDARDS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

MCC6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fraction, e.g. by using visual fraction models and equations to represent the problem.

Compute fluently with multi-digit numbers and find common factors and multiples.

MCC6.NS.2 Fluently divide multi-digit numbers using the standard algorithm.

MCC6.NS.3 Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

MCC6.NS.4 Find the greatest common factor of two whole numbers less than or equal to 100. And the least common multiple of two whole numbers less than or equal to 1. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no remainder**.**

STANDARDS FOR MATHEMATICAL PRACTICE

1. Make sense of problems and persevere in solving them. Students understand the problem context in order to translate them into ratios/rates.
2. Reason abstractly and quantitatively. Students understand the relationship between two quantities in order to express them mathematically. They use ratio and rate notation as well as visual models and contexts to demonstrate reasoning.
3. Construct viable arguments and critique the reasoning of others. Students construct and critique arguments regarding appropriateness of representations given ratio and rate contexts. For example, does a tape diagram adequately represent a given ratio scenario.
4. Model with mathematics. Students can model problem situations symbolically (tables, expressions or equations), visually (graphs or diagrams) and contextually to form real-world connections.
5. Use appropriate tools strategically. Students choose appropriate models for a given situation, including tables, expressions or equations, tape diagrams, number line models, etc.
6. Attend to precision. Students use and interpret mathematical language to make sense of ratios and rates.
7. Look for and make use of structure. The structure of a ratio is unique and can be used across a wide variety of problem-solving situations. For instance, students recognize patterns that exist in ratio tables, including both the additive and multiplicative properties. In addition, students use their knowledge of the structures of word problems to make sense of real-world problems.

8. Look for and express regularity in repeated reasoning. Students utilize repeated reasoning by applying their knowledge of ratio, rate and problem solving structures to new contexts. Students can generalize the relationship between representations, understanding that all formats represent the same ratio or rate.

ENDURING UNDERSTANDINGS

- A ratio is a number that relates two quantities or measures within a given situation in a multiplicative relationship (in contrast to a difference or additive relationship).The relationships and rules that govern whole numbers, govern all rational numbers.
- Making explicit the type of relationships that exist between two values will minimize confusion between multiplicative and additive situations.
- Ratios can express comparisons of a part to whole, (a/b with $b \neq 0$), for example, the ratio of the number of boys in a class to the number of students in the class.
- The ratio of the length to the width of a rectangle is a part-to-part relationship.
- Understand that fractions are also part-whole ratios, meaning fractions are also ratios. Percentages are ratios and are sometimes used to express ratios.
- Both part-to-whole and part-to-part ratios compare two measures of the same type of thing. A ratio can also be a rate.
- A rate is a comparison of the measures of two different things or quantities; the measuring unit is different for each value. For example if 4 similar vans carry 36 passengers, then the comparison of 4 vans to 36 passengers is a ratio.
- All rates of speed are ratios that compare distance to time, such as driving at 45 miles per hour or jogging at 7 minutes per mile.
- Ratios use division to represent relations between two quantities.

CONCEPTS & SKILLS TO MAINTAIN

It is expected that students will have prior knowledge/experience related to the concepts and skills identified below. It may be necessary to pre-assess in order to determine if time needs to be spent on conceptual activities that help students develop a deeper understanding of these ideas.

- Multiples and Factors
- Divisibility Rules
- Relationships and rules for multiplication and division of whole numbers as they apply to decimal fractions
- Understanding of common fractions

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for middle school students. **Note – Different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks.**

http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary

http://intermath.coe.uga.edu/dictnary/homepg.asp
Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

http://www.corestandards.org/Math/Content/mathematics-glossary/glossary

- **Percent:** A fraction or ratio in which the denominator is 100. A number compared to 100.

- **Proportion:** An equation which states that two ratios are equal.

- **Rate:** A comparison of two quantities that have different units of measure

- **Ratio:** compares quantities that share a fixed, multiplicative relationship.

- **Rational number:** A number that can be written as a/b where a and b are integers, but b is not equal to 0.

- **Tape diagram:** A thinking tool use to visually represent a mathematical problem and transform the words into an appropriate numerical operation. Tape diagrams are drawings that look like a segment of tape, used to illustrate number relationships. Also known as Singapore Strips, strip diagrams, bar models or graphs, fraction strips, or length models

- **Unit Ratio:** are ratios written as some number to 1.

- **Quantity:** is an amount that can be counted or measured.

MISCONCEPTIONS

Often there is a misunderstanding that a percent is always a natural number less than or equal to 100. Provide examples of percent amounts that are greater than 100%, and percent amounts that are less 1%.

FORMATIVE ASSESSMENT LESSONS (FALs)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on types of Formative Assessment Lessons may be found in the Comprehensive Course Guide.

Name_____

TASK: RATIOS AND RATES

A ratio can be expressed three ways:

- Using the fraction bar as in $\frac{2}{3}$
- Using a colon symbol as in 2:3
- Using the word "to" as in 2 to 3.

Write each ratio using the other two ways:

1. The ratio of 3 inches to 20 feet.

2. The ratio of 26 students: 1 class

3. The ratio of $\frac{2\ boys}{3\ girls}$

When the denominator of a rate is 1, we call the rate a **unit rate**. We usually use the key word **per** or the division symbol (/)to indicate a unit rate. For example: If a student earns $7.65 per hour, it is the same as $7.65/hour, and means $7.65 for every hour of work.

Find the unit rate for the following:

4. 120 eggs from 20 chickens

5. $55 for 20 people

6. 250 miles in 4 hours

7. 60 feet in 4 minutes

8. 48 books for 16 students

9. 56 children from 14 families

Unit rates can also be used to solve problems.

10. Which is the better deal: 8 ounces of shampoo for $0.89 or 12 ounces for $1.47

11. Which is the better deal: 3 cans of soda for $1.27 or 5 cans of soda for $1.79

12. Which is the better deal: 10 pounds of hamburger for $4.99 or 5 pounds of hamburger for $2.69

13. Which is traveling faster: Traveling 300 miles in 5 hours or traveling 250 miles in 4 hours

14. Which is traveling faster: Traveling 75 miles in 1 hour or traveling 280 miles in 3.5 hours

15. Which is traveling faster: Traveling 150 yards in 40 seconds or traveling 406 feet in 35 seconds

Name:_____

TASK: CONSTANT DIMENSIONS

1. Measure the rectangle using five different units and record in the chart below.

Unit	**Length**	**Width**
Inches		
Centimeter		

2. Create a list of ordered pairs to represent the measurements of the rectangle you found (L, W)

3. Create a scatter plot of measurements. Use graph paper.

4. Do the points appear to be random, or do they seem to follow a pattern? Explain.

5. If someone used gumballs to measure the length and width, and their ordered pair were placed at (22, 10), would we suspect they made a good measurement? What if the ordered pair had the coordinates (16, 10.5) would that be reasonable? What is your reasoning?

6. The length of the rectangle measured approximately 7.9 nickels. What is the width of the rectangle in nickels?

7. If the length of the rectangle is 15 wooches, determine the width of the rectangle in wooches.

Name_____

TASK: HOW MANY NOSES ARE IN YOUR ARMS

1. Measure your arm span from finger tip to finger tip. Measure your height. Find the ratio of your arm span to your height.

2. Measure the length of your foot and the distance around your fist. Find the ratio of the length of your foot to the distance around your fist.

3. Using the picture of the Statue of Liberty and the fact that her nose measures 4 feet 6 inches from the bridge to the tip, determine the length of the Statue of Liberty's right arm, the one holding the torch.

4. What strategy did you use to determine the length of the Statue of Liberty's right arm?

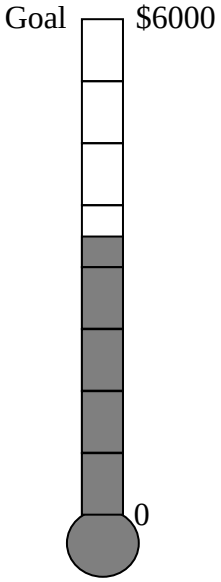
5. Is the ratio of the measurement of the length of your nose to the length of your arm the same as the ratio of the Statue of Liberty's?

Name_____

TASK: REACHING THE GOAL

After looking at the scale to the right…

Michael said, "We have reached 5/8 of our goal."
Juan said, "I think we have earned about 60% of the $6,000 we need."
Fiona said, "We have earned about $3,500."
Nathan said, "You are all close, but none of you are correct."



Nathan is correct.

a) Represent the amount earned as a fraction, as a decimal, as a percent, and as a dollar amount.

b) Show how you know that the *fraction*, the *percent*, the *decimal*, and the *dollar amount* answers are all equivalent in this situation.

c) Which 3 of the amounts are *always* equivalent to each other? Why?

d) Which amount is *not always* equivalent to the others? Why not?

Name_____
Task: Free Throws

1. Juan made 13 out of 20 free throws. If Bonita shoots 25 free throws, what's the minimum number she has to make in order to have a better free-throw percentage than Juan?

2. Juan continues to shoot free throws. How many free throws would Juan need to make out of fifty to have the same percentage that Bonita now has?

3. Can Bonita continue to have the same percentage if she shoots 60 free throws? If she shoots 75 free throws? If yes, how many free throws does she need to make?

FORMATIVE ASSESSMENT LESSON: SHARING COSTS- TRAVELING TO SCHOOL

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1366

TASK COMMENTS

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Sharing Costs: Traveling to School*, is a Formative Assessment Lesson (FAL) that can be found at the
website: http://map.mathshell.org/materials/lessons.php?taskid=489&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
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STANDARDS FOR MATHEMATICAL PRACTICE

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2. Reason abstractly and quantitatively.
4. Model with mathematics.

FORMATIVE ASSESSMENT LESSON: OPTIMIZING- SECURITY CAMERAS

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1354

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Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Optimizing: Security Cameras*, is a Formative Assessment Lesson (FAL) that can be found at the
website: http://map.mathshell.org/materials/lessons.php?taskid=482&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
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Name_____

TASK: ICE CREAM OR CAKE?

Suppose you survey all the students at your school to find out whether they like ice cream or cake better as a dessert, and you record your results in the contingency table below.

	ice cream	**cake**	**totals**
boys	82	63	145
girls	85	70	155
totals	167	133	300

a) What percentage of students at your school prefers ice cream over cake?

b) At your school, are those preferring ice cream more likely to be boys or girls?

c) At your school, are girls more likely to choose ice cream over cake than boys are?